

1106-05-2023

Dennis Eichhorn*, Department of Mathematics, University of California, Irvine, Irvine, CA 92697, and **Felix Breuer** and **Brandt Kronholm**. *The Combinatorics Governing the Periodicity of $p(n, d)$ Modulo M .*

Since the generating function for $p(n, d)$, the number of partitions of n into parts of size at most d , is a rational function, we know that $p(n, d)$ is periodic modulo M . Can one find a purely combinatorial explanation for the periodicity?

The search for an explanation led us to study the geometry of lattice points in polyhedra, which ultimately inspired a new decomposition of partitions into their “ ℓ -box remainder” and “ ℓ -box quotient.” These two new objects bear many similarities to the ℓ -core and ℓ -quotient of a partition, which were used in the famous combinatorial proof of the first four Ramanujan congruences by Garvan, Kim, and Stanton. This new ℓ -box decomposition does lead to a combinatorial proof of the periodicity of $p(n, d)$ modulo M , and the proof provides substantial structural information about the behavior of $p(n, d)$.

Some immediate consequences of this work include new proofs of several infinite families of known Ramanujan-type congruences for $p(n, d)$. Furthermore, these methods apply equally well to partitions whose parts come from any fixed finite set A , which allows for many new generalizations of the previously known infinite families of congruences. (Received September 15, 2014)