1106-05-2040 **Salvatore Tringali*** (salvo.tringali@gmail.com), Texas A&M University PO Box 23874, Doha, Qatar. *On the structure of sets of lengths in η-systems.* Preliminary report.

Let $\Phi = (\mathbb{S}, \|\cdot\|, \mathfrak{P}, \Theta, \mathcal{E})$ be a (natural) η-system, i.e., $\mathbb{S} = (S, \cdot)$ is a semigroup, $\|\cdot\|$ is a seminorm $\mathbb{S} \to (\mathbf{N}, +, \le)$, $\mathfrak{P}$ and Θ are subsets of S with Θ disjoint from the subsemigroup generated by $\mathfrak{P}$, and $\mathcal{E}$ is an equivalence on $\mathfrak{P}^\dagger$ (here, $X^\dagger$ means the Kleene plus of a set X).

Let σ be the factorization map $S^\dagger \to S : (x_1, \ldots, x_n) \mapsto x_1 \cdots x_n$. For each $k \in \mathbf{N}^+$ we let $\mathcal{U}_k$ be the set of all integers l for which there exist *minimal* Φ-factorizations $\mathfrak{p} = (p_1, \ldots, p_m), \mathfrak{q} = (q_1, \ldots, q_n) \in \mathfrak{P}^\dagger$ such that $\sigma(\mathfrak{p}) = \sigma(\mathfrak{q})$, $\sum_{i=1}^m \|p_i\| = k$ and $\sum_{i=1}^n \|q_i\| = l$. We show that this generalizes sets of lengths, as defined in the factorization theory of monoids, to η-systems.

We say that Φ is atomic if $\Theta \cup \sigma(\mathfrak{P}^\dagger) = S$, and smoothly bounded if $\varrho_k := \sup(\mathcal{U}_k) < \infty$ for each k and $\sup_{k\ge1}(\varrho_{k+1} - \varrho_k) < \infty$. We give examples of [atomic] smoothly bounded η-systems and prove that if Φ is smoothly bounded then the $\mathcal{U}_k$'s have a non-trivial additive structure. (Received September 16, 2014)