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Ira M. Gessel* (gessel@brandeis.edu) and **Yan Zhuang**. *Counting permutations with even valleys and odd peaks.*

We find the exponential generating function for permutations with all valleys even and all peaks odd, answering a question posed by Liviu Nicolaescu. The generating function is

$$\left(1 - E_1x + E_3\frac{x^3}{3!} - E_4\frac{x^4}{4!} + E_6\frac{x^6}{6!} - E_7\frac{x^7}{7!} + \cdots\right)^{-1}, \quad (1)$$

where $\sum_{n=0}^{\infty} E_n x^n/n! = \sec x + \tan x$, which resembles David and Barton's generating function

$$\left(1 - x + \frac{x^3}{3!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \cdots\right)^{-1}, \quad (2)$$

for permutations with no increasing runs of length 3 or more.

Following Dennis Chebikin, we define an alternating descent of a permutation to be an odd descent or an even ascent, and we define an alternating run to be a maximal consecutive subsequence with no alternating descents. Then the permutations we want to count are those with no alternating runs of length 3 or more.

Using noncommutative symmetric functions, we explain the similarity of (1) and (2) as a special case of a very general connection between generating functions for permutations by increasing runs and by alternating runs. (Received September 03, 2014)