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Levent Alpoge*, Harvard University, Cambridge, MA. *The average elliptic curve has few integral points.*

Counting the number of integer solutions to $y^2 = x^3 + Ax + B$ is an old, old problem — Fermat discussed the cases of $A = 0$, $B = -2, -4$ when first presenting his method of descent, for instance. Siegel proved that such an equation can only have finitely many integral solutions given A and B (so long the equation actually gives an elliptic curve). How many should there be if we vary A and B ? On average, the answer should be zero. I will show it is bounded and give an explicit upper bound. The methods will combine the recent results of Bhargava-Shankar on averaging Selmer groups, the Kabatiansky-Levenshtein bound from the theory of sphere packing, Roth's theorem in Diophantine approximation, and the observation that integral points tend to repel in the Mordell-Weil lattice (much like rational points on higher genus curves repel according to Mumford's gap principle). If time permits I will also mention similar results for thinner families: $y^2 = x^3 + B$, $y^2 = x^3 + Ax$, and the congruent number curves $y^2 = x^3 - D^2x$. (Received September 16, 2014)