1106-11-2946 **Jared S Weinstein*** (jsweinst@math.bu.edu), Boston University, Department of Mathematics and Statistics, Boston, MA 02215. *Exploring the Galois group of the rational numbers: recent breakthroughs.*

It's a basic result that the polynomial $x^2 + 1$ factors modulo an odd prime p exactly when $p \equiv 1 \pmod{4}$. Are there such rules governing the factorization of any polynomial modulo a prime? This question lies at the heart of algebraic number theory, which has roots extending back to Fermat and Gauss, and which is best phrased in terms of the Galois group of the rationals, $\operatorname{Gal}(\overline{\mathbf{Q}}/\mathbf{Q})$. There are (mostly open) conjectures which link this problem to the behavior of analytic objects known as automorphic forms. We will discuss these and also a recent breakthrough of Peter Scholze, who found a surprising connection between $\operatorname{Gal}(\overline{\mathbf{Q}}/\mathbf{Q})$ and the geometry of certain manifolds, called locally symmetric spaces. (Received September 17, 2014)