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Daniel A. Spielman* (daniel.spielman@yale.edu). *Graphs, Vectors and Matrices*.

We will explain how we use linear algebra to understand graphs and how recently developed ideas in graph theory have inspired progress in linear algebra.

Graphs can take many forms, from social networks to road networks, and from protein interaction networks to scientific meshes. One of the most effective ways to make sense of the diverse structure a graph can display is to study algebraic properties of a matrix associated with a graph. We will see some of what can be learned from studying the Laplacian matrix of a graph.

Many graph algorithms and analyses are simplified by the approximation of a graph by a simpler graph. We will examine the problem of sparsifying a graph—approximating it by a graph on the same vertices, but with fewer edges. For example, the best sparse approximations of complete graphs are provided by the famous Ramanujan graphs. As the Laplacian matrix of a graph is a sum of outer products of vectors, one for each edge, the problem of sparsifying a general graph can be recast as a problem of approximating a collection of vectors by a small subset of those vectors. The resulting problem appears similar to the problem of Kadison and Singer in Operator Theory. We will sketch how research on the sparsification of graphs led to its solution. (Received September 15, 2014)