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Ron Gould, Victor Larsen* (vlarsen@emory.edu) and **Luke Postle**. *Asymptotic density of k -critical graphs.*

A graph G is k -critical if $\chi(G) = k$ and every proper subgraph $H \subsetneq G$ has $\chi(H) \leq k - 1$. Thus, a k -critical graph can be viewed as a minimal k -chromatic graph. A natural question about a minimal k -chromatic graph is how small such a graph can be. For a fixed number of vertices, n , let $f_k(n)$ denote the minimum number of edges in a k -critical graph on n vertices. The Hájos construction (and its generalization, the Ore construction) implies that $f_k(n+k-1) \leq f_k(n) + (k-1) \left(\frac{k}{2} - \frac{1}{k-1} \right)$. Therefore, the asymptotic density φ_k of a k -critical graph as we increase the number of vertices is at most $\frac{k}{2} - \frac{1}{k-1}$. In a 2012 paper, Kostochka and Yancey were able to confirm that $\varphi_k = \frac{k}{2} - \frac{1}{k-1}$, by providing a lower bound on $f_k(n)$. We examine the graphs that attain this bound (the k -extremal graphs) and identify key subgraphs. This allows us to obtain an asymptotic density of $\varphi_k + \epsilon$ on k -critical graphs that do not contain these subgraphs. (Received September 13, 2014)