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Lower-order biases in elliptic curve Fourier coefficients. Preliminary report.

Let $\mathcal{E} : y^2 = x^3 + A(T)x + B(T)$ be a nontrivial one-parameter family of elliptic curves over $\mathbb{Q}(T)$, with $A(T), B(T) \in \mathbb{Z}(T)$, and consider the k th moments $A_{k,\mathcal{E}}(p) := \sum_{t \bmod p} a_{\mathcal{E}_t}(p)^k$ of the Fourier coefficients $a_{\mathcal{E}_t}(p) := p + 1 - |\mathcal{E}_t(\mathbb{F}_p)|$. Rosen and Silverman proved a conjecture of Nagao relating the first moment $A_{1,\mathcal{E}}(p)$ to the rank of the family over $\mathbb{Q}(T)$, and Michel proved the second moment is $A_{2,\mathcal{E}}(p) = p^2 + O(p^{3/2})$. Cohomological arguments show the lower order terms are of sizes $p^{3/2}$, p , $p^{1/2}$, and 1. In every case we are able to analyze, the largest lower order term that does not average to zero is on average negative. We prove this “bias conjecture” for several large classes of families, including families with rank, complex multiplication, and unusual distributions of signs. We identify all lower order terms in large classes of families, shedding light on the objects controlling these terms. The negative bias in these terms has implications toward the excess rank conjecture and the behavior of zeros near the central point of elliptic curve L -functions. (Received September 15, 2014)