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**Turnage-Butterbaugh and Madeleine Weinstein.** *Benfordness of Zeckendorf Decomposition.*

Zeckendorf showed that every positive integer can be decomposed uniquely into a sum of non-consecutive Fibonacci numbers. Additionally, the Fibonacci numbers are known to satisfy Benford's law of digit bias, which means that the density of elements with first digit  $d$  is  $\log_{10}(1 + \frac{1}{d})$ . According to this law, the smaller the digit, the more likely it is to occur as a leading digit. Thus the number 1 occurs as a leading digit about 30% of the time, while the number 9 occurs about 4.5% of the time.

We prove that for a randomly selected integer between 1 and the  $n$ th Fibonacci number, as  $n \rightarrow \infty$  the leading digits of the Fibonacci summands in its Zeckendorf decomposition are arbitrarily close to Benford almost surely. The proof proceeds by first analyzing random subsets of Fibonacci numbers for Benfordness. The main ingredient there is showing sets of density are preserved under this process. Using this, we solve our stated problem by proving a correspondence between Zeckendorf decompositions and random subsets of Fibonacci numbers. In those sets the Fibonacci numbers are chosen with a probability  $p = 1/\varphi^2$  (where  $\varphi$  is the golden mean) if the previous Fibonacci number wasn't chosen, and 0 otherwise. (Received September 16, 2014)