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**Chi Huynh** (nhuynh30@gatech.edu), **Carsten Peterson** (carsten.peterson@yale.edu) and **Yen Nhi Truong Vu\*** (ytruongvu17@amherst.edu). *On Summand Minimality of Generalized Zeckendorf Decompositions.*

Zeckendorf states that every number can be uniquely represented as a sum of non-consecutive Fibonacci numbers, paralleling a base  $d$  representation. Given a recurrence  $a_n = c_1 a_{n-1} + \dots + c_t a_{n-t}$ , the tuple  $(c_1, \dots, c_t)$  is called the signature. Miller and Wang, and independently Hamlin, proved that given a non-negative linear recurrence with  $c_1 \geq 1$ , each number has a unique representation with respect to the recurrence sequence, called the generalized Zeckendorf decomposition (GZD), composed of “digits” from an allowable finite list. We prove for all  $n$ , the GZD uses the fewest summands of all representations if and only if the signature is weakly decreasing. To prove sufficiency, we construct an algorithm to arrive at any number’s unique GZD and show the number of summands decreases over the algorithm’s course. To prove necessity we handle a few distinct cases. When  $c_1 > 1$ , we give an example of a non-GZD representation of a number that has fewer summands than the GZD. When  $c_1 = 1$ , we non-constructively prove the existence of a counterexample using the irreducibility of certain polynomials and growth rate arguments. Joint work with Katherine Cordwell, Max Hlavacek, Steven J. Miller and Eyvindur A. Palsson. (Received September 19, 2016)