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This is a proposed UGR project on some plausible Diophantine-analytic identities as $\frac{\sin x}{x}$ converges to 1 along certain values $x \rightarrow 0$. Using $\csc^2 x = \frac{1}{x^2} + \frac{1}{3} + \frac{x^2}{15} + \mathcal{O}(x^4)$ about 0, as $m \rightarrow \infty$ one has $\frac{1}{4} \csc^2 \frac{\pi}{2^{m+2}} = \frac{4^{m+1}}{\pi^2} + \frac{1}{12} + \frac{\pi^2}{15 \times 4^{m+3}} + \mathcal{O}(4^{-2m})$. We wrote this with $\frac{1}{4}$ and $m+2$ on the left to go with $\frac{1}{2 - \underbrace{\sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}}}_{m \text{ times}}}} = \frac{1}{4} \csc^2 \frac{\pi}{2^{m+2}}$. We ask whether

$$\lfloor \frac{1}{4} \csc^2 \frac{\pi}{2^{m+2}} \rfloor =? \lfloor \frac{4^{m+1}}{\pi^2} + \frac{1}{12} \rfloor (?)$$

for all m . We similarly raise the questions $\lfloor \frac{1}{4} \csc^2 \frac{3\pi}{2^{m+3}} \rfloor =? \lfloor \frac{4^{m+2}}{9\pi^2} + \frac{1}{12} \rfloor (?)$, $\lfloor \frac{1}{4} \csc^2 \frac{\pi}{3 \times 2^{m+2}} \rfloor =? \lfloor \frac{9 \times 4^{m+1}}{\pi^2} + \frac{1}{12} \rfloor (?)$, and $\lfloor \frac{1}{4} \csc^2 \frac{5\pi}{12 \times 2^m} \rfloor =? \lfloor \frac{9 \times 4^{m+1}}{25\pi^2} + \frac{1}{12} \rfloor (?)$.

[Using Mathematica, we verified all four when $m \leq 20,000$.] (Received September 21, 2016)