

1125-41-2904

Dylan Airey* (dylan.airey@utexas.edu), **Steve Jackson** and **Bill Mance**. *Some complexity results in the theory of normal numbers.*

Let $\mathcal{N}(b)$ be the set of real numbers which are normal to base b . A well-known result of Ki and Linton is that $\mathcal{N}(b)$ is $\mathbf{\Pi}_3^0$ -complete. We show that the set $\mathcal{N}^\perp(b)$ of reals y which preserve $\mathcal{N}(b)$ under addition is also $\mathbf{\Pi}_3^0$ -complete. We use the characterization of $\mathcal{N}^\perp(b)$ given by Rauzy in terms of an entropy-like quantity called the noise. It follows from our results that no further characterization theorems could result in a still better bound on the complexity of $\mathcal{N}^\perp(b)$. We compute the exact descriptive complexity of other naturally occurring sets associated with noise. One of these is complete at the $\mathbf{\Pi}_4^0$ level. Finally, we get upper and lower bounds on the Hausdorff dimension of the level sets associated to the noise. (Received September 20, 2016)