1125-46-3 **Tepper L. Gill*** (tgill@howard.edu), DC. *Non-uniqueness of the dual of a Banach space and its application.*

Let $\mathcal{B}$ be a Banach space with a shrinking S-basis. All studies assume the properties of $\mathcal{B}^*$, the dual, are unique. I show that $\mathcal{B}^*$ has three different representations $\mathcal{B}_d^*$, $\mathcal{B}_h^*$ and $\mathcal{B}_s^*$ bijectively related to $\mathcal{B}$. $\mathcal{B}_d^*$ is the set of duality maps and $\mathcal{B}_h^* \leftrightarrow \mathcal{B}$ is a (conjugate) isometric isomorphism. If the S-basis is not shrinking, $\mathcal{B}_h^*$ and $\mathcal{B}_s^*$ are subspaces of $\mathcal{B}^*$. Banach asked if a separable Banach space had a Schauder or S-Basis. Mazur proved that, every separable Banach space always contains an infinite dimensional subspace with a S-basis. In 1972 Enflo showed the existence of a separable Banach space with out an S-basis. Our first application shows that, there exists Hilbert spaces with $\mathcal{H}_1 \subset \mathcal{B} \subset \mathcal{H}_2$ as dense continuous embeddings. Markushevich gave a weaker definition of a basis (M-basis) and proved that every separable Banach space B always has one. We give a positive answer to the norm one (M-basis) problem. (Received September 08, 2016)