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William M. Higdon* (wmhigdon@butler.edu), Department of Mathematics and Statistics, 4600 Sunset Avenue, Indianapolis, IN 46208. *On The Numerical Ranges of Composition Operators Induced By Mappings With The Denjoy-Wolff Point On The Boundary.*

The *numerical range* of an operator T on a Hilbert space H , denoted $W(T)$, is the set,

$$W(T) = \{ \langle Tx, x \rangle \mid x \in H \text{ and } \|x\| = 1 \}.$$

Elementary properties of $W(T)$ include that it is a bounded subset of \mathbf{C} , it is convex, it contains the eigenvalues of T , and, more generally, its closure includes the spectrum of T . The work presented here answers the question posed by Professors Paul S. Bourdon and Joel H. Shapiro in their paper (“When Is Zero In The Numerical Range Of A Composition Operator?”, J. IEOT., 44 (2002), 410-441):

“Suppose the symbol ϕ of the composition operator C_ϕ on $H^2(D)$ is univalent, not linear fractional, and is of parabolic nonautomorphism type. Is $0 \in W(C_\phi)$?”

The answer has been, in general, unknown. One property of such a mapping is that it has derivative equal to 1 at its Denjoy-Wolff point (boundary fixed point). Using elementary properties of C_ϕ -invariant subspaces, the answer to the question is demonstrated. Together with the work of Bourdon and Shapiro, this provides a complete description of when 0 belongs to the numerical range of a composition operator C_ϕ on $H^2(D)$. (Received September 15, 2016)