In recent years, there has been a growing interest in the study of nonlinear differential equations with multiple solutions. In this talk, I will present a two-level algorithm based on spectral methods and regeneration homotopy to solve the second order elliptic equation with nonlinearity of polynomial type. Both theoretical and numerical results verify that the proposed method enjoys the following merits: (i) it guarantees multiple solutions; (ii) the computational cost is relatively small; (iii) it is of high-order accuracy. (Received September 07, 2016)