Geometric patterns comprise a key component in architecture of the Islamic world. The reasons for this emphasis in the arts are complex; here it is argued that geometric patterns may visually express new mathematical ideas. Between the 9th and 12th centuries major advances involved radical developments in algebra, trigonometry, spherical geometry, optics, number theory, and possibly algebraic geometry. Mathematicians included al-Khwarezmi and his successors, Omar Khayyam, Sharaf al-Din al-Tusi, and Nasir al-Din al-Tusi, among others. My research on monuments in Iran and neighboring regions (Iraq, Afghanistan, Azerbaijan, Syria, Turkey) from an art historical perspective delves into circles and centers, points of intersection, lines and angles, axes and grids, symmetry and tessellations, periodic and quasiperiodic patterns. This paper focuses on the use of intersecting polygons to tile the plane; through visual analysis we may recognize intensive explorations into the nature of two-dimensional space. Such innovations in mathematics and architecture occurred where skill, knowledge, and patronage converged in intellectual centers such as Baghdad, Isfahan, Ghazna, Damascus, Aleppo, and Maragha, with profound implications relating the histories of Islamic art and mathematics. (Received September 08, 2015)