The Second Incompleteness Theorem actually makes 2 assertions:

1. $\text{Con}_\Sigma$ states that $\Sigma$ is consistent;
2. $\Sigma \nvdash \text{Con}_\Sigma$ if $\Sigma \supseteq \Sigma_{\text{PA}}$ is consistent.

(1) had no explicit definiens.

If (1) is—as the definiendum, lacking another statement of place, suggests—related to (the theory of) $\Sigma$, then, as we will show, (2) implies non (1), whence (1) & (2) becomes a contradiction in terms. In addition, the generalisation: $\kappa$ states consistency, cannot be fulfilled at all.

If $\Sigma$ is decidable, (1) becomes true in $\text{Th} (\mathcal{N})$, the deductively inaccessible theory of arithmetics.

More innately, $\kappa$ states that $\Sigma$ is consistent :iff $\Sigma \nvdash \kappa$. Consequently, if $\Sigma$ is consistent, all of the then existing $\kappa$, unprovable from $\Sigma$, state this, and, if $\Sigma$ is inconsistent, no $\kappa$ states that $\Sigma$ is consistent. If (1) is interpreted in this way, (1) follows from (2), but $\text{Con}_\Sigma$ is not distinguished from any other $\Sigma \nvdash \kappa$.

Compare the ASL abstract. Joint work with Wilfried Buchholz. (Received September 23, 2015)