We study the tensor square conjecture, which states that for all $n$ with a few exceptions, there are irreducible representations of the symmetric groups $S_n$ whose tensor squares contain every irreducible representation. Our main result is that tensor 4th powers suffice to contain all these representations for large enough $n$. We also show that tensor squares of certain representations contain almost all irreducibles with respect to two natural probability distributions. Our main tool is the semigroup property, which allows breaking partitions down into smaller cases that can be analyzed using simpler methods. (Received September 19, 2015)