We establish a combinatorial connection between the real geometry and the $K$-theory of complex Schubert curves $S(\lambda \cdot)$, which are one-dimensional Schubert problems defined with respect to flags osculating the rational normal curve. Recent work by Speyer and by the second author showed that the real geometry of these curves is described by the orbits of a map $\omega$ on skew tableaux, defined as the commutator of jeu de taquin rectification and promotion. In particular, the real locus of the Schubert curve is naturally a covering space of $\mathbb{RP}^1$, with $\omega$ as the monodromy operator.

We provide a local algorithm for computing $\omega$ without rectifying the skew tableau, and show that certain steps in our algorithm are in bijective correspondence with Pechenik and Yong’s genomic tableaux, which enumerate the $K$-theory Littlewood-Richardson coefficient of the Schubert curve. We then give purely combinatorial proofs of several numerical results relating the $K$-theory and the real geometry of $S(\lambda \cdot)$. (Received September 21, 2015)