The recursive structure of genus polynomials for linear families.

The genus polynomial for a finite graph $G$ is the generating function $g_G(z) = \sum a_i z^i$, where $a_i$ is the number of imbeddings of $G$ in the surface of genus $i$. A linear family $G_n$ of graphs is formed by taking $n$ copies of the same graph $G$ and forming a path of them by adding edges in the same way between one copy of $G$ and the next; Stahl suggested these families as a case study for genus polynomials. For any such linear family there is a production or transfer matrix $M(z)$ and initial vector $v(z)$ (all entries are polynomials in $z$ with non-negative integer coefficients) such that the genus polynomial for $G_n$ is $M^n(z)v(z)$. These matrices have been computed by hand for a few small cases and by computer for some larger examples. Almost nothing is known about the entries of $M(z)$ or the behavior of $M^n(z)$. We conjecture that for sufficiently large $n$ all entries of $M^n(z)$ are non-zero. This would imply that the Markov chain associated with $M(1)$ is regular, allowing one to infer the long-run distribution of the different types of imbeddings of $G_n$. (Received September 21, 2015)