Given a graph $G = (V, E)$, and a given function $\alpha : V \to \mathbb{N}$, an $\alpha$-orientation is an orientation of the edges such that the out-degree of each vertex $v$ equals $\alpha(v)$. S. Felsner (TU-Berlin) in 2004 proved that the set of alpha-orientation on an embedded planar graph (a planar map) carries the structure of a distributive lattice, with unique maximal and minimal elements. He uses this result, for example, to construct canonical spanning trees on rooted planar maps as well as several other canonical structures on planar maps.

We obtain a generalization of Felsner’s result to higher genus orientable surfaces with possible application to bijective methods in map enumeration and construction. Additionally, by applying this result to pairs of Cayley maps (strongly symmetric embeddings of Cayley graphs) we obtain potential applications to the study of finite group extensions. (Received September 22, 2015)