Let $L : V(G) \to \mathbb{Z}$ be a one-to-one integer labeling of the vertices of a simple graph $G$.

(1) We call $L$ a **prime distance labeling** of $G$ if for any two adjacent vertices $u$ and $v$, the integer $|L(u) - L(v)|$ is prime; in this case, we say $G$ is a **prime distance graph**.

(2) We call $L$ a **$k$-prime product distance labeling** of $G$ if for any two adjacent vertices $u, v$, the integer $|L(u) - L(v)|$ is a product of at most $k$ (not necessarily distinct) primes; in this case, we say $G$ is a **$k$-prime power distance graph**. If $G$ has a $k$-prime product distance labeling and not a $(k - 1)$-prime product distance labeling, then we set $\pi(G) = k$.

(3) We call $L$ a **prime power distance labeling** of $G$ if for any two adjacent vertices $u$ and $v$, the integer $|L(u) - L(v)|$ is a positive power of a prime; in this case, we say $G$ is a **prime power distance graph**.

In this paper, we characterize some families of prime distance graphs, prime power distance graphs, and $k$-prime product distance graphs; provide bounds for $\pi(G)$; and make connections between graphs of these kinds and several important theorems and conjectures from Number Theory (including the Green-Tao Theorem and Fermat’s Last Theorem). (Received September 09, 2015)