Let $E$ be an elliptic curve defined over a number field $F$. It is a classical theorem of Mordell and Weil that the collection of points of $E$ with coordinates in $F$ form a finitely generated abelian group. We seek to understand the subgroup of points with finite order. In particular, given a positive integer $d$, we would like to know precisely which abelian groups arise as the torsion subgroup of an elliptic curve defined over a number field of degree $d$, and we would like to know how the size of the torsion subgroup grows as $d$ increases. I will discuss recent progress on these problems for the special class of elliptic curves with complex multiplication under the assumption that $d$ is odd. (Received September 19, 2015)