We present a new method of proving the Diophantine extremality of various dynamically defined measures, vastly expanding the class of measures known to be extremal. Extremal measures are locally finite Borel measures on $\mathbb{R}^d$ for which most points are not very well approximable by ones with rational coordinates. Our work generalizes and improves the celebrated theorem of Kleinbock and Margulis (’98) resolving Sprindzuk’s conjecture, as well as its extension by Kleinbock, Lindenstrauss, and Weiss (’04), hereafter abbreviated KLW. The key technical idea, which has led to a plethora of new applications, is a significant weakening of KLW’s sufficient conditions for extremality.

As applications we prove that Patterson–Sullivan measures of all nonplanar geometrically finite groups, and that Gibbs measures of nonplanar infinite iterated function systems (including those which do not satisfy the open set condition) and rational functions are quasi-decaying. This research is part of an ongoing collaboration with Lior Fishman (North Texas), David Simmons (York) and Mariusz Urbański (North Texas). (Received September 05, 2015)