Let $C$ be an Artin-Schreier curve of genus $g$ over a finite field $k := \mathbb{F}_q$ of characteristic $p$. A number of authors have considered a weighted sum of $C$ using the automorphism group of $C$ over $k$, mostly in the cases when $p = 2$. For odd $p$, we consider a closely related weighted sum for $C$ over finite fields of all characteristics $p$. Each Artin-Schreier curve $C$ has an associated rational equation $y^p - y = u(x)$ for $u(x) \in k(x)$ with a set of $n$ poles. In the process of determining the weighted sums, we consider the action of $\text{PGL}_2(k)$ on these rational equations and their poles. In particular, the number of orbits of these $n$-sets is known if the field of definition is $k$ (López and Nart), and we find the number of orbits for appropriate field extensions of $k$ in our cases. (Received September 15, 2015)