It is well-known [?] that for a divisible ordered Abelian group $G$, and a field $K$ that is algebraically closed, or real closed, the Hahn field $K((G))$ is also algebraically closed, or real closed. The ideas go back to Newton and Puiseux. Each element $r$ of $K((G))$ is a generalized power series with terms corresponding to elements of a well-ordered subset of $G$ and with coefficients in $K$. The length of $r$ is the order type of the set of $g \in G$ with non-zero coefficient. We give a technical theorem, for the case where $G$ is Archimedean, bounding the length of a root $r$ of a polynomial $p(x)$ in terms of the lengths of the coefficients in $p(x)$. To obtain the technical theorem, we follow unpublished notes of Starchenko, adding further ordinal calculations.

Using the technical theorem, we can prove the conjecture from [?], stated in Lange’s talk.

References


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