A spectrahedral cone $C$ is a slice of the cone of positive semidefinite matrices with a linear subspace $L$. The ranks of extreme rays of spectrahedral cones have been a subject of extensive study. It is natural to ask for what subspaces $L$ do all of the extreme rays of $C$ have rank 1? When $L$ is a union of coordinate subspaces the answer was given by Agler-Helton-McCullough-Rodman. It turns out that this question has an unexpected connection to algebraic geometry and we will present some steps toward a full classification of such spectrahedral cones based on the classification of small reduced schemes by Eisenbud-Green-Hulek-Popescu. This is joint work with Rainer Sinn and Mauricio Velasco. (Received September 16, 2015)