The (Iwahori-)Hecke algebra in the title is a $q$-deformation $\mathcal{H}$ of the group algebra of a finite Weyl group $W$. The algebra $\mathcal{H}$ has a natural enlargement to an endomorphism algebra $A = \text{End}_\mathcal{H}(\mathcal{T})$ where $\mathcal{T}$ is a $q$-permutation module. In type $A_n$ (i.e., $W \cong S_{n+1}$), the algebra $A$ is a $q$-Schur algebra which is quasi-hereditary and plays an important role in the modular representation of the finite groups of Lie type. In other types, $A$ is not always quasi-hereditary, but the authors conjectured in 1998 that $\mathcal{T}$ can be enlarged to an $\mathcal{H}$-module $\mathcal{T}^+$ so that $A^+ = \text{End}_\mathcal{H}(\mathcal{T}^+)$ is at least standardly stratified, a weaker condition than being quasi-hereditary, but with “strata” corresponding to Kazhdan-Lusztig two-sided cells.

The main result of this paper is a first step toward this conjecture, a “local” version in the equal parameter case, localizing at cyclotomic polynomials with some restrictions, and using the theory of rational Cherednik algebras. As time permits, I will try to place the result in a broader context. (Received September 17, 2015)