Let $T$ be an operator on a Hilbert space $H$ with numerical radius $w(T) \leq 1$. According to a theorem of Berger and Stampfli, if $f$ is a function in the disk algebra such that $f(0) = 0$, then $w(f(T)) \leq \|f\|_{\infty}$. We give a new and elementary proof of this result using finite Blaschke products.

A well-known result relating numerical radius and norm says $\|T\| \leq 2w(T)$. We obtain a local improvement of this estimate, namely, if $w(T) \leq 1$ then

$$\|Tx\|^2 \leq 2 + 2\sqrt{1 - |\langle Tx, x \rangle|^2} \quad (x \in H, \|x\| \leq 1).$$

Using this refinement, we give a simplified proof of Drury’s teardrop theorem, which extends the Berger–Stampfli theorem to the case $f(0) \neq 0$.

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