We consider hyperbolicity preserving operators with respect to a new linear operator representation on $\mathbb{R}[x]$. In essence, we demonstrate that every Hermite and Laguerre multiplier sequence can be diagonalized into a sum of hyperbolicity preserving operators, where each of the summands forms a classical multiplier sequence. Interestingly, this does not work for other orthogonal bases; for example, this property fails for the Legendre basis. We establish many new formulas concerning the $Q_k$’s of Peetre’s 1959 differential representation for linear operators in the specific case of Hermite and Laguerre diagonal differential operators. Additionally, we provide a new algebraic characterization of the Hermite multiplier sequences and also extend a recent result of T. Forgács and A. Piotrowski on hyperbolicity properties of the polynomial coefficients in hyperbolicity preserving Hermite diagonal differential operators. (Received September 08, 2015)