Some of the most important problems in geometric partial differential equations are related to the understanding of singularities. This usually happens through a blow up procedure near the potential singularity which uses the scaling properties of the equation. In the case of a parabolic equation the blow up analysis often leads to special solutions which are defined for all time $-\infty < t \leq T$, for some $T \leq +\infty$. We refer to them as ancient if $T < +\infty$ and eternal if $T = +\infty$. The classification of such solutions, when possible, often sheds new insight to the singularity analysis.

We will give a survey of recent research progress on ancient solutions to geometric flows such as the Ricci flow, the Mean Curvature flow and the Yamabe flow. Our discussion will also include other models of nonlinear parabolic partial differential equations.

We will address both Liouville type results for the classification of ancient or eternal solutions to parabolic equations as well as the construction of new ancient solutions from the gluing of two or more solitons. (Received September 10, 2015)