It is not yet known if the global attractor of the space periodic 2D Navier-Stokes equations contains non-stationary solutions $u(x,t)$ such that their energy and enstrophy per unit mass, are constant for every $t$. The study of the properties of such solutions was initiated in, where, due to the hypothetical existence of such solutions, they were called “ghost solutions”. In this work, we introduce and study geometric structures shared by all ghost solutions. This study led us to consider a subclass of ghost solutions for which those geometric structures have a supplementary stability property. In particular, we found constraints on the wave vectors of the active modes of the latter ghost solutions. In particular, we show that the wave vectors of the active modes of this subclass of ghost solutions must satisfy certain supplementary constraints. We also found a computational way to check for the existence of these ghost solutions. By using the Finite Volume Method in the software OpenFoam, we implement a method to solve the Navier-Stokes equations. Some numerical simulations as well as theoretic results are given. (Received August 05, 2015)