Analytical Combinatorics brings to bear methods of complex function theory on the asymptotic analysis of enumerative generating functions. In recent times this subject has been gaining in popularity and applicability motivated by other developments in mathematics and computer science such as the study of random matrices and expander graphs. In this talk we will discuss some further extensions of this line of study that arise through analytical dynamics related to the combinatorics of maps. In particular we will focus on problems related to two aspects of four-valent maps: geodesic distance on the sphere and the asymptotics of generating functions for higher genus maps. We will show how both of these problems are connected to the same family of combinatorial dynamical systems which are in fact Hamiltonian in nature, a feature that is key in this study. (Received September 22, 2015)