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*Iterations for the lemniscate constant resembling the Archimedean algorithm for  $\pi$ .*

The lemniscate constant  $L = 2.62205755429212\dots$  is one half the perimeter of the unit lemniscate curve just as  $\pi$  is one half the perimeter of the unit circle. It is well known that the Archimedean iterative algorithm

$$(1) \ a(n+1) = 2a(n)b(n)/(a(n) + b(n)) \text{ and}$$

$$(2) \ b(n+1) = \sqrt{a(n+1)b(n)} \text{ with initial values } a(1) = 4 \text{ and } b(1) = 2\sqrt{2} \text{ converges to } \pi.$$

In this paper we show that the iterative algorithm

$$(3) \ a(n+1) = 2a(n)a(n)/(a(n) + b(n)) \text{ and}$$

$$(4) \ b(n+1) = \sqrt{a(n+1)b(n)}$$

with initial values  $a(1) = 4$  and  $b(1) = 2\sqrt{2}$  converges to the lemniscate constant  $L$ .

Notice that the only change is that  $b(n)$  in (1) has been replaced by  $a(n)$  in (3). (2) is exactly the same as (4) and the initial values are the same. The derivation is based on an infinite product of nested radicals given in recent years by Aaron Levin ([1] and [2]) that resembles the famous product of nested radicals for  $\pi$  by Vieta.

[1] Levin, A., A New Class of Infinite Products Generalizing Viète's Product Formula for  $\pi$ , *The Ramanujan Journal*, 10(2005), pp. 305–324.

[2] Levin, A., A Geometric Interpretation of an Infinite Product for the Lemniscate Constant, *The American Mathematical Monthly*, Vol. 113, No. 6 (Jun. - Jul., 2006), pp. 510-520 (Received September 22, 2015)