Xiaoyue Cui* (cuix.wsu@gmail.com), 727 W Martin Luther King Drive, Apt. 514W, Cincinnati, OH 45220, and Guozhen Lu. New Characterizations of High Order Sobolev Space on Euclidean Spaces.

The main purpose of this paper is to study some new characterizations of the high order Sobolev spaces $W^{m,p}(\mathbb{R}^N)$. Actually, we will present here two types of characterizations: by $m$-th differences and by $m$-Taylor reminder. Therefore, two of our main results are as follows: If $f \in L^p(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$, then

1. $f \in W^{m,p}(\mathbb{R}^N)$ if and only if
   \[
   \sup_{0<\delta<1} \int_{\mathbb{R}^N} \frac{\delta^p}{|x-y|^{N+mp}} \, dx \, dy < \infty.
   \]

2. $f \in W^{m,p}(\mathbb{R}^N)$ if and only if
   \[
   \sup_{0<\delta<1} \int_{\mathbb{R}^N} \int_{\mathbb{R}^N} \frac{\delta^p}{|x-y|^{N+mp}} \, dx \, dy < \infty, \quad f \in W^{m-1,p}(\mathbb{R}^N)
   \]

(Received August 19, 2015)