Where does Kepler’s third law hold?

A theorem of Gromov asserts that the only homogeneous Riemannian manifolds admitting dilations are Euclidean spaces. We explain the surprising relationship between this theorem and Kepler’s third law of planetary motion. Kepler’s first two laws are known to hold in spherical and hyperbolic geometries, while the third law fails. Gromov tells us why, and where to look for geometries which may admit a version of the third law. We introduce the Kepler problem on the Heisenberg group, the simplest Carnot group, and state the Kepler-Heisenberg third law. Time permitting, we explore other interesting properties of this system, including the existence of periodic orbits and the near integrability of the dynamics. (Received September 22, 2015)