In joint work with Peter Albers, Joel Fish, and Katrin Wehrheim, we study the Piunikhin-Salamon-Schwarz-map from Morse homology to Floer homology. It is classically defined for semi-positive symplectic manifolds by counting pseudoholomorphic curves. We extend these definitions to morphisms $PSS : HM \to HF$ and $SSP : HF \to HM$ for general closed symplectic manifolds using a polyfold description of the moduli spaces and abstract transversality results. The polyfolds are built as fiber products of the symplectic field theory polyfolds and compactified Morse moduli spaces. Cobordism and grading arguments then prove that the composition of $SSP$ with $PSS$ is an isomorphism on Morse homology with Novikov coefficients. While there is no such cobordism for the reversed composition, the previous suffices to reprove the Arnold conjecture, bounding the number of Hamiltonian orbits below by the total rank of homology, which was previously proven by virtual moduli cycle techniques. Moreover, we outline further refined polyfold techniques which will prove that $PSS$ is indeed an isomorphism with inverse $SSP$. (Received September 13, 2015)