The Catalan numbers are a sequence of positive integers that provide answers to certain combinatorial questions. For example, in how many ways can a polygon be cut into triangles? Other types of problems also lead to the Catalan numbers: counting binary trees, balancing parentheses, finding paths through a grid, shaking hands in a circle, etc. I will present visualizations of several solution sets related to the Catalan numbers, and show how connections can be found to map one set into another. Furthermore, I will show how my artistic work has drawn on these visualizations and relationships. (Received September 20, 2015)