The simplest but still most important among nonlinear eigenvalue problems are the polynomial eigenvalue problems (PEPs). Any PEP is associated with a matrix polynomial \( P(\lambda) = \sum P_i \lambda^i \), where \( P_i \in \mathbb{C}^{m \times n} \). The standard way to numerically solve a PEP is to linearize the matrix polynomial \( P(\lambda) \) into a matrix pencil \( L(\lambda) = \lambda Y + X \). A matrix pencil is said to be a strong linearization of \( P(\lambda) \) if they share the same finite and infinite eigenstructure, which may be computed using any of the well-known algorithms for solving generalized eigenvalue problems. In this work we introduce a new set of linearizations that allows one to design matrix polynomial eigensolvers that are easy to implement, are valid regardless of whether the matrix polynomial is regular or singular, allow one to compute all the eigenstructure of the matrix polynomial, exploit any structure that the matrix polynomial might posses, and include error and condition estimates. These desirable properties allow one to overcome all the drawbacks presented in previous methods, making this linearizations more attractive than the ones known so far. (Received September 22, 2015)