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Pittsburgh, PA 15213. *The Cantor-Bernstein Theorem: How many proofs?*

The Cantor-Bernstein Theorem states: if there is an injection  $f$  from  $A$  to  $B$  and an injection  $g$  from  $B$  to  $A$ , then there is a bijection  $h$  from  $A$  to  $B$ . Dedekind was the first to prove the Theorem – without appealing to Cantor’s well-ordering principle, but rather by using his chain theory; the proof was only published in 1932. A careful analysis of his proof reveals a basic structure of argumentation that can be seen in the many other proofs. (A full account of proofs and their history is given in Hinkis’ book, *Proofs of the Cantor-Bernstein Theorem*, 2013.) My contention is that there is essentially a single proof, in two minor variants. The second variant is due to Zermelo in 1908 who used, not knowing about Dedekind’s proof, the latter’s chain theory. It is of interest to note that the various proofs obtain one of two bijections. The proofs have been fully and “naturally” formalized, by Sieg and Walsh, in Zermelo Fraenkel set theory ZF. That allows us to pinpoint exactly where proof differences appear. (Received September 09, 2017)