Theorems in extremal graph theory ask to optimize a combinatorial invariant over a fixed family of graphs. In this talk, we discuss how to prove several theorems in this area where the graph invariant in question is a function of the eigenvalues or eigenvectors of the adjacency matrix of the graph. A representative result is a proof of a conjecture of Boots and Royle from 1991: the planar graph of maximum spectral radius (of its adjacency matrix) is the join of an edge and a path. This is joint work with Josh Tobin. (Received September 20, 2017)