A balanced incomplete block design (BIBD(v, k, λ)) \((V, B)\) is a combinatorial design in which (i) \(|V| = v\), (ii) for each block \(B \in B\), \(|B| = k\), and (iii) each 2-subset of \(V\) occurs in precisely \(λ\) blocks of \(B\). A BIBD\((v, 3, 2)\) is a twofold triple system (TTS\((v)\)).

Given a combinatorial design \(D\) with block set \(B\), the block-intersection graph (BIG) of \(D\) is the graph having \(B\) as its vertex set, and in which two vertices \(B_1 \in B\) and \(B_2 \in B\) are adjacent if and only if \(|B_1 \cap B_2| \neq 0\). Similarly, the \(i\)-block-intersection graph (i-BIG) of \(D\) is the graph having \(B\) as its vertex set, and in which two vertices \(B_1 \in B\) and \(B_2 \in B\) are adjacent if and only if \(|B_1 \cap B_2| = i\).

Recently David Pike and I completed the spectrum for TTSs with (respectively, without) Hamilton cycles in their 2-BIGs. In this talk I will present some of the constructions that collectively lead to these results. (Received September 22, 2017)