To detect statistical anomalies in networks, we compare empirically observed networks with realizations from a realistic random graph model. Since many real world networks exhibit degree heterogeneity, we consider some challenges in randomly constructing graphs with a given bidegree sequence in an unbiased way. In particular, we propose a novel method for the asymptotic enumeration of directed graphs that realize a bidegree sequence, $d$, with maximum degree $d_{max} = O(S^{\frac{1}{2}} - \tau)$ for an arbitrarily small positive number $\tau$, where $S$ is the number of edges specified by $d$; the previous best results allow for $d_{max} = o(S^{\frac{1}{2}})$. Our approach is based on two key steps, graph partitioning and degree preserving switches. The former allows us to relate enumeration results to degree sequences that are easy to handle, while the latter facilitates expansions based on numbers of shared neighbors of pairs of nodes. (Received September 25, 2017)