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Eric Rowland* (eric.rowland@hofstra.edu). *Enumeration of binomial coefficients by their p -adic valuations.*

In 1947 Nathan Fine obtained a product formula for the number of integers m in the range $0 \leq m \leq n$ such that $\binom{n}{m}$ is not divisible by p . Subsequently, many authors found formulas counting binomial coefficients with p -adic valuation $\nu_p(\binom{n}{m}) = \alpha$ for particular values of p^α , but a general formula remained elusive. We give a matrix product, generalizing Fine's result, for the generating function

$$T_p(n, x) := \sum_{m=0}^n x^{\nu_p(\binom{n}{m})}$$

which simultaneously counts binomial coefficients with p -adic valuation α for all $\alpha \geq 0$. The polynomial $T_p(n, x)$ was recently identified by Spiegelhofer and Wallner as a central object in the structure of formulas for the number of binomial coefficients with p -adic valuation α . We also give a further generalization to multinomial coefficients. (Received August 13, 2017)