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Joshua D Brakensiek*, Department of Mathematical Sciences, Wean Hall 6113, Carnegie Mellon University, Pittsburgh, PA 15213. *Vertex Isoperimetry and Independent Set Stability for Tensor Powers of Cliques.*

The n th tensor power of t -clique, K_t^n , is the graph on vertex set $[t]^n = \{1, \dots, t\}^n$ such that two vertices $x, y \in [t]^n$ are connected iff $x_i \neq y_i$ for all $i \in [n]$. Let the density of a subset S of K_t^n be $\mu(S) = |S|/t^n$. Let the vertex boundary of a set S be the vertices, including those of S , which are incident to some vertex of S .

First, given $\nu \in [0, 1]$, what is the smallest possible density of the vertex boundary of a subset of density ν ? Let $\Phi_t(\nu)$ be the infimum of these minimum densities as $n \rightarrow \infty$. We find a recursive relation to compute $\Phi_t(\nu)$ efficiently to arbitrary precision.

Second, given an independent set I of density $(1 - \epsilon)/t$, how close it is to a maximum density independent set J ? We show that $\mu(I \setminus J) \leq 4\epsilon^{(\log t)/(\log t - \log(t-1))}$ as long as $\epsilon < 1 - 3/t + 2/t^2$. This substantially improves on results of Alon, Dinur, Friedgut, and Sudakov (2004) and Ghandehari and Hatami (2008) which had an $O(\epsilon)$ upper bound. We also show the exponent is optimal as $n \rightarrow \infty$ and $\epsilon \rightarrow 0$. The methods have similarity to recent work by Ellis, Keller, and Lifshitz (2016) in the context of Kneser graphs and other settings. (Received June 21, 2017)