Finding three-term progressions in subsets.

How big can a subset of \{1, 2, \ldots, N\} or an abelian group be if it does not contain any three-term progression \(x, x+y, x+2y\) with \(y \neq 0\)? How big can a subset of the alternating group be if it does not contain any three-term progression \(x, xy, xy^2\) with \(y\) not equal to the identity permutation? How big can a subset of \{1, 2, \ldots, N\} or \(\mathbb{F}_q\) be if it does not contain any three-term progression \(x, x + y^{1060}, x + y^{31} + y^{26} + y^{23} + y^{14} + y^{10}\) with \(y \neq 0\)? In this talk, I will discuss the relationship between these three questions, the techniques used in trying to answer them, and the different challenges encountered when we consider longer progressions. (Received September 26, 2017)