1135-05-38 **Federico Ardila***, San Francisco State University, San Francisco, CA 94110, and **Marcelo Aguiar**, Cornell University, Ithaca, NY 14853. *Algebraic structures on polytopes.*

Generalized permutahedra are a beautiful family of polytopes with a rich combinatorial structure and strong connections to optimization. We study their algebraic structure: we prove they are the universal family of polyhedra with a certain "Hopf monoid" structure. This construction provides a unifying framework to organize and study many combinatorial families: 1. It uniformly answers open questions and recovers known results about graphs, posets, matroids, hypergraphs, and simplicial complexes. 2. It reveals that three combinatorial reciprocity theorems of Stanley and Billera–Jia–Reiner on graphs, posets, and matroids are really the same theorem. 3. It shows that permutahedra and associahedra "know" how to compute the multiplicative and compositional inverses of power series. The talk will be accessible to undergraduates and will not assume previous knowledge of these topics. (Received June 27, 2017)