We discuss a new connection between knot and graph theory arising from the origami method of nanoscale DNA self-assembly design, in which a long unknotted scaffolding strand of DNA is folded into the shape of an embedded graph by smaller staple strands. For graphs embedded on surfaces, optimal routes for the scaffolding strand correspond to unknotted A-trails, i.e. Eulerian circuits with certain turning restrictions. We show that every Eulerian graph has an embedding containing an unknotted A-trail and that there exist embedded Eulerian graphs with no unknotted A-trails. On the torus, we characterize when checkerboard-colorable embeddings have unknotted A-trails and apply this characterization to certain regular triangulations. In closing, we discuss new directions arising from the case of graphs in space but not on surfaces. (Received September 06, 2017)