For a general family of graphs on $\mathbb{Z}^n$, we translate the edge-isoperimetric problem into a continuous isoperimetric problem in $\mathbb{R}^n$. We then solve the continuous isoperimetric problem using the Brunn-Minkowski inequality and Minkowski’s theorem on Mixed Volumes. This translation allows us to conclude, under a reasonable assumption about the discrete problem, that the shapes of the optimal sets in the discrete problem approach the shape of the optimal set in the continuous problem as the size of the set grows. The solution is the zonotope defined as the Minkowski sum of the edges of the original graph.

We demonstrate the efficacy of this method by revisiting some previously solved classical edge-isoperimetric problems. We then apply our method to some discrete isoperimetric problems which had not previously been solved. The complexity of those solutions suggest that it would be quite difficult to find them using discrete methods only. (Received July 12, 2017)