Combinatorial geometry in origami: the Kawasaki-Maekawa connection.

The study of flat origami involves the properties and processes of folding two-dimensional materials using straight crease lines so that the image of the folding is also two-dimensional. That is, the folded object is flat and can be pressed in a book without crumpling or adding new creases. There are two fundamental results that pertain to the interior vertices in a flat origami crease pattern. One is geometrical: Kawasaki’s Theorem states that a vertex will fold flat if and only if the alternating sum of the sector angles around the vertex is zero. The other is combinatorial: Maekawa’s Theorem states that the difference between the number of mountain (convex) and valley (concave) at a flat-foldable vertex is always 2. It turns out that these results, Maekawa’s Theorem and the necessary direction of Kawasaki’s Theorem, are equivalent if we generalize their domains from folding flat paper to folding cones. Doing this also expands the two Theorems to statements about the geometry of the folded vertex. We will explain and prove this more general result and discuss the possibility of generalizing to multiple-vertex crease patterns. (Received September 17, 2017)